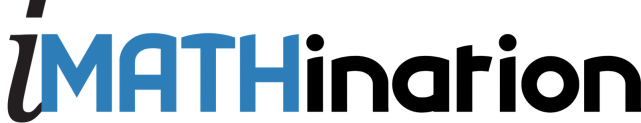
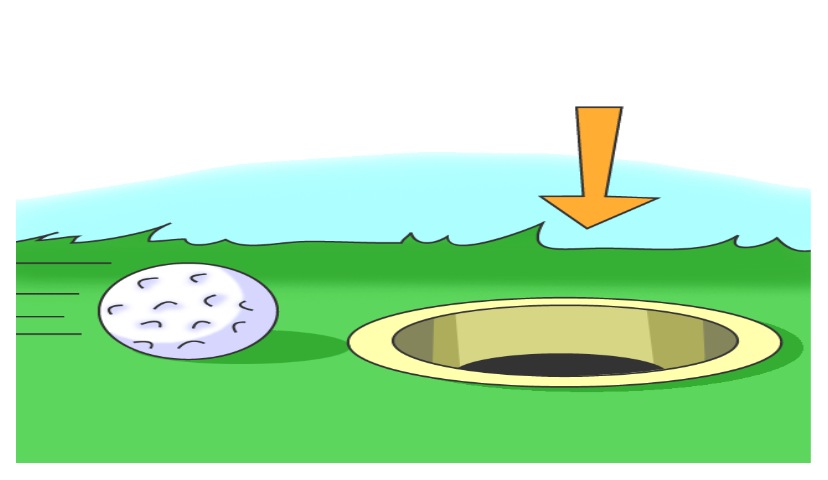
**Transform Your Classroom and Construct Learning with Geometry Golf**

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2012 Conference



**Common Core State Standards for Mathematics**

Practice Standards: This lesson can address all 8 of the Math Practice Standards

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1**. Make sense of problems and persevere in solving them.**

Mathematically proficient students:

* explain to themselves the meaning of a problem and looking for entry points to its solution.
* analyze givens, constraints, relationships, and goals.
* make conjectures about the form and meaning of the solution attempt.
* consider analogous problems, and try special cases and simpler forms of the original problem.
* monitor and evaluate their progress and change course if necessary.
* draw diagrams of important features and relationships, graph data, and search for regularity or trends.
* use concrete objects or pictures to help conceptualize and solve a problem.
* check their answers to problems using a different method.
* ask themselves, “Does this make sense?”
* understand the approaches of others to solving complex problems.

2. **Reason abstractly and quantitatively.**

Mathematically proficient students:

* make sense of quantities and their relationships in problem situations.
* *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
* *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).

1. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students:

* understand and use stated assumptions, definitions, and previously established results in constructing arguments.
* analyze situations by breaking them into cases
* justify their conclusions, communicate them to others, and respond to the arguments of others.
* compare the effectiveness of plausible arguments
* distinguish correct logic or reasoning from that which is flawed
* elementary students construct arguments using objects, drawings, diagrams, and actions..
* later students learn to determine domains to which an argument applies.
* listen or read the arguments of others, decide whether they make sense, and ask useful questions

1. **Model with mathematics.**

Mathematically proficient students:

* apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
* simplify a complicated situation, realizing that these may need revision later.
* identify important quantities in a practical situation
* map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
* analyze those relationships mathematically to draw conclusions.
* interpret their mathematical results in the context of the situation.
* reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

1. **Use appropriate tools strategically.**

Mathematically proficient students

* consider available tools when solving a mathematical problem.
* are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
* detect possible errors by using estimations and other mathematical knowledge.
* know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
* identify relevant mathematical resources and use them to pose or solve problems.
* use technological tools to explore and deepen their understanding of concepts.

1. **Attend to precision.**

Mathematically proficient students:

* try to communicate precisely to others.
  + - use clear definitions in discussion with others and in their own reasoning. .
    - specify units of measure and label axes to clarify the correspondence with quantities in a problem.

1. **Look for and make use of structure.**

Mathematically proficient students:

* look closely to discern a pattern or structure.
* step back for an overview and can shift perspective.
* see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

1. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students:

* notice if calculations are repeated
* look both for general methods and for shortcuts.
* maintain oversight of the process, while attending to the details.
* continually evaluate the reasonableness of intermediate results.

Common Core State Standards for Mathematics – Content Standards

|  |  |
| --- | --- |
| Congruence | * Experiment with transformations in the plane. G.CO.1, 2, 3, 4, 5 * Understand congruence in terms of rigid motions. *Build on rigid motions as a familiar starting point for development of concept of geometric proof.* G.CO.6, 7, 8 * Prove geometric theorems. *Focus on validity of underlying reasoning while using variety of ways of writing proofs* G.CO.9, 10, 11 * Make geometric constructions. *Formalize and explain processes G.CO.12, 13* |
| Similarity, Right Triangles, and Trigonometry | * Understand similarity in terms of similarity transformations. G.SRT.1a, 1b, 2, 3 * Define trigonometric ratios and solve problems involving right triangles. G.SRT.6, 7, 8 * Apply trigonometry to general triangles. G.SRT.9, 10, 11 |
| Expressing Geometric Properties with Equations | * Use coordinates to prove simple geometric theorems algebraically. Include distance formula; relate to Pythagorean theorem G. GPE.4, 5, 6, 7 |
| Modeling with Geometry | * Apply geometric concepts in modeling situations. G.MG.1, 2, 3 |

Miniature Golf

How to get a Hole-in-One  
MCj03657320000[1]Objective: Find a path for the ball to follow to make a hole-in one using reflections to find angles of incidence equal to angles of reflection.

Materials: You’ll need paper, tape, a compass and straightedge or a mira (sometimes called a geo reflector, geo reflecta, or something similar)

Banking the ball off of one wall **–** make sure to have a large enough piece of paper to construct/find the points of reflection --- be ready to tape another piece of paper to the original one if you need it!

1. Locate the hole (H) & beginning position of the ball (B).

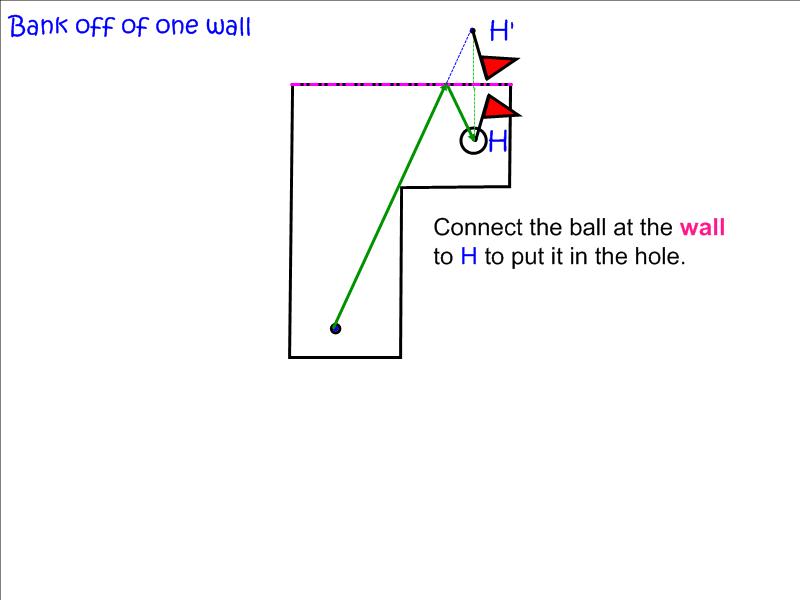
2. Pick a wall to bank the ball off of and label it W1.

3. With a compass or mira, reflect the hole (H) over wall 1 and mark it H1.

4. With a straight edge, connect the ball (B) with H1 using a dotted line to locate where on the wall the ball needs to reflect off of.

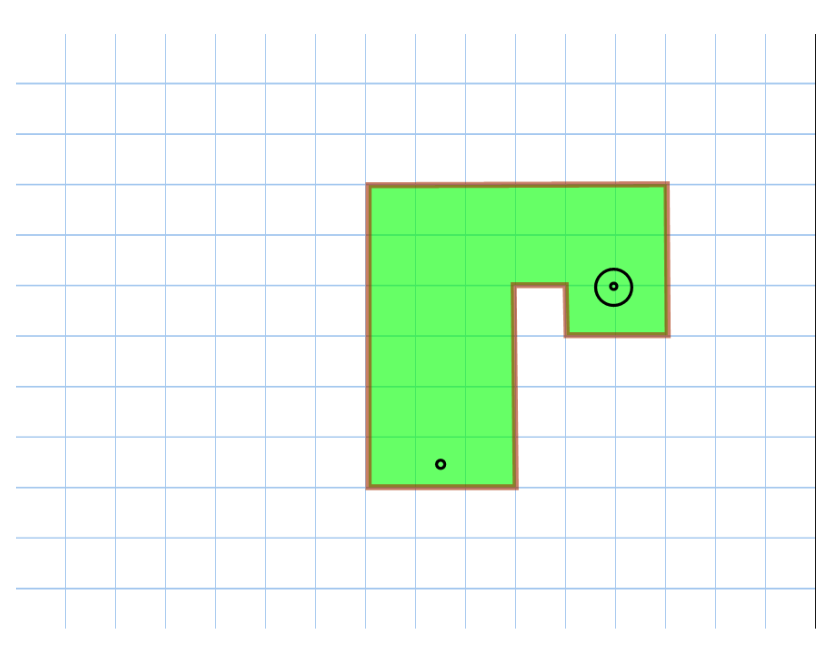
5. Connect the hole (H) to the point of intersection of the wall and the line from the ball to H1 to complete the path of the ball. The solid lines show the path the ball will take to make the hole in one. Realize that speed of the ball, the type of surface used on the hole, the accuracy of the golfer, and other factors may influence the actual success of the hole-in one! (See the diagrams below showing two examples of the same golf hole but banking the ball off of different walls.)

Example 1:This is close to the corner, but if you are a good golfer, you can get a hole-in-one.

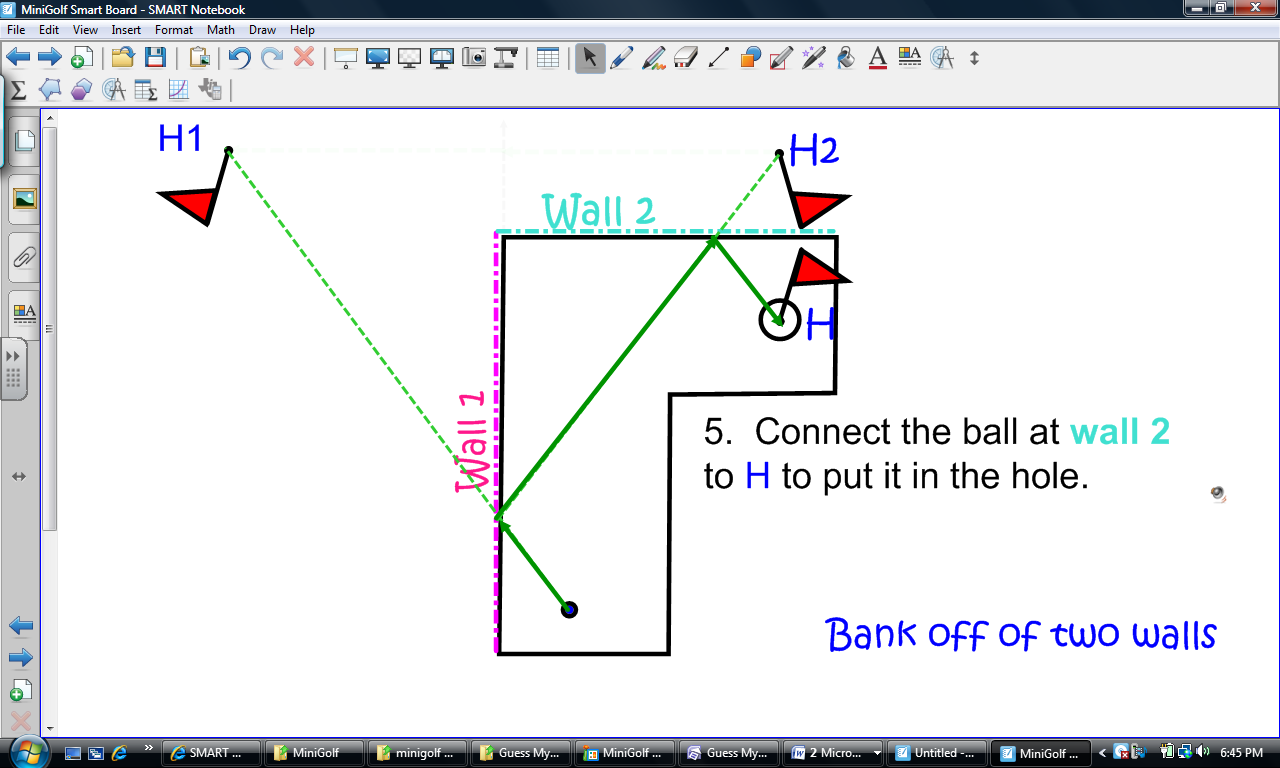


**Banking the ball off of 2 walls**

(\*Note: not all choices of walls will result in possible holes-in-one because of the design of the hole. If you begin connecting points and the points of contact do not end up on the correct wall, that path is not possible and other walls should be chosen or the placement of the ball or the hole must be changed.)

Let’s try finding a path first on a hole plotted on graph paper…

1. Locate the hole (H) & beginning position of the ball (B).
2. Pick and label the walls starting at the hole and working backwards from the hole. Mark the walls W2 for the second wall the ball will hit before landing in the hole, and W1 for the first wall to bank it off to send it to the second wall. (in this case the first wall hit by the ball).
3. With a compass or mira, reflect the hole (H) over the wall 2 (W2 on this example) and mark it H2. You may have to extend a wall to be able to complete a reflection.
4. With a compass or mira, reflect H2 over wall 1 (W1) and mark it H1.
5. With a straight edge, connect the ball (B) with H1 to locate where on the wall the ball needs to hit. Then connect that point of contact/intersection with H2 to locate where on wall 2 the ball needs to hit.
6. Finally, connect the point of contact/intersection on the wall 2 with the hole (H) to complete the path of the ball. (See the diagram.)
7. Notice that at each point of contact with the wall the angle of incidence = angle of reflection! (BUT, don’t make the mistake that ALL of the angles at those points are NOT the same measures!)

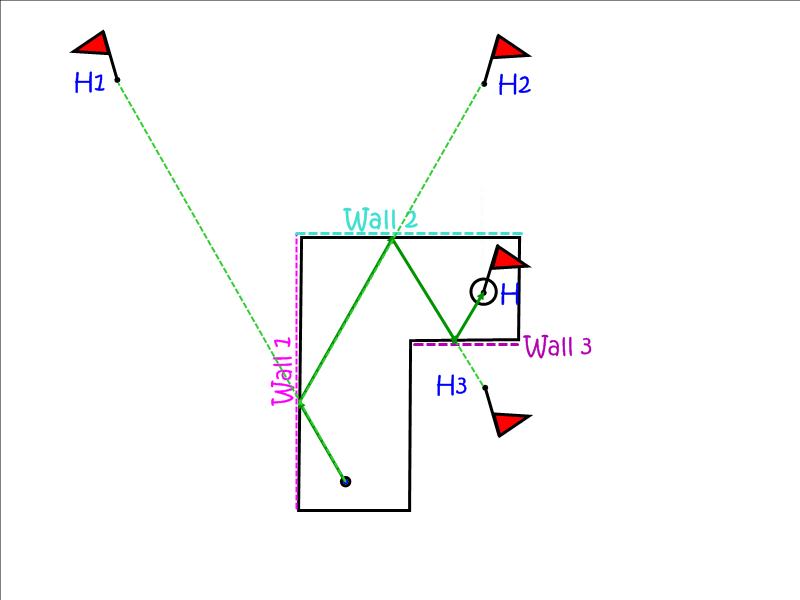


## Banking the ball off of 3 or more walls

(\*Note: not all choices of walls will result in possible holes-in-one because of the design of the hole. If you begin connecting points and the points of contact do not end up on a wall, that path is not possible and other walls should be chosen or the placement of the ball or the hole must be changed.)

1. Locate the hole (H) & beginning position of the ball (B).

2. Pick and label the walls starting at the hole and working backwards from the hole. Mark the walls W1 for the first wall the ball will hit before landing in the hole, W2 for the next wall, W3 for the next wall to hit, etc.

3. With a compass or mira, reflect the hole (H) over wall with the highest number and mark it H# (this example: H3 if you going to bank it off of 3 walls, H4 if you’re planning to hit 4 walls, etc.). You may have to extend a wall to be able to complete a reflection. 

4. With a compass or mira, reflect H’ over wall 2 (W2) and mark it H’2. Then reflect H2 over   
wall 1 (W1) and label it H1.

5. With a straight edge, connect the ball (B) with H1 to locate where on the wall the ball needs to hit. Then connect that point of contact/intersection with H2 to locate where on wall 2 the ball needs to hit. Then continue until all of the walls chosen have points of contact/intesection.

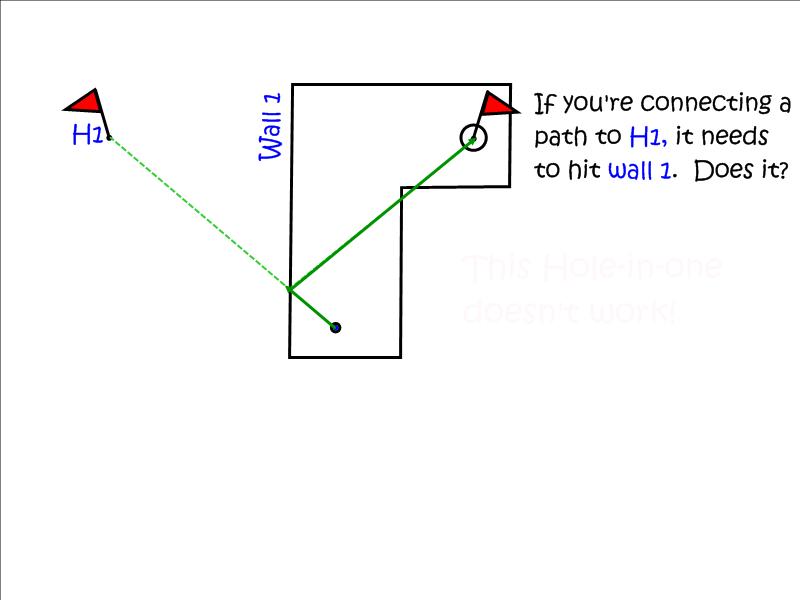
6. Finally, connect the point of contact on the last wall with the hole (H) to complete the path of the ball. (See the diagram.)

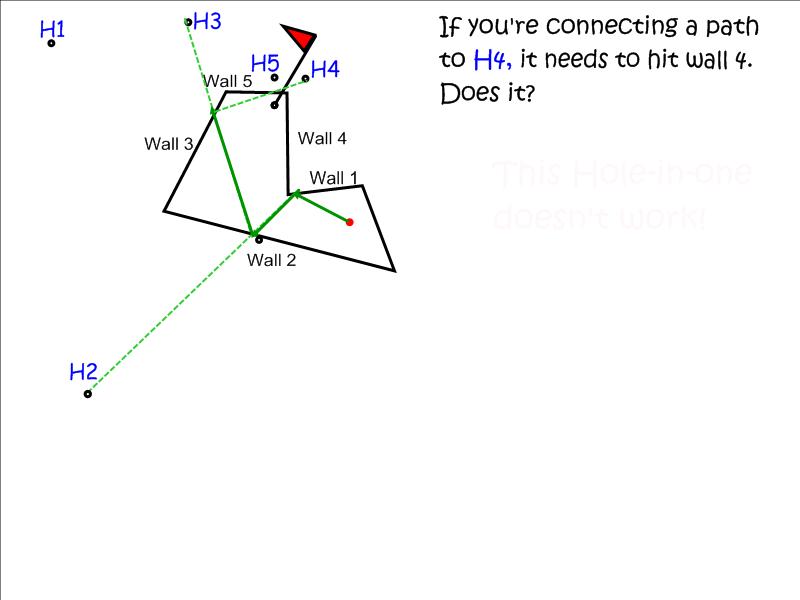
7. Notice that at each point of contact with the wall the angle of incidence = angle of reflection! (BUT, don’t make the mistake that ALL of the angles at those points are NOT the same measures!)

NOTE: If you want to place obstacles on the hole, do it AFTER you find the path of the ball to make sure that the obstacle doesn’t get in the way of the ball and the path it needs to follow.

This diagram shows how to make a hole-in-one by banking the ball off of 5 walls. Notice that at each point of contact with the wall the angle of incidence = angle of reflection! (BUT, don’t make the mistake that ALL of the angles at those points are NOT the same measures!)

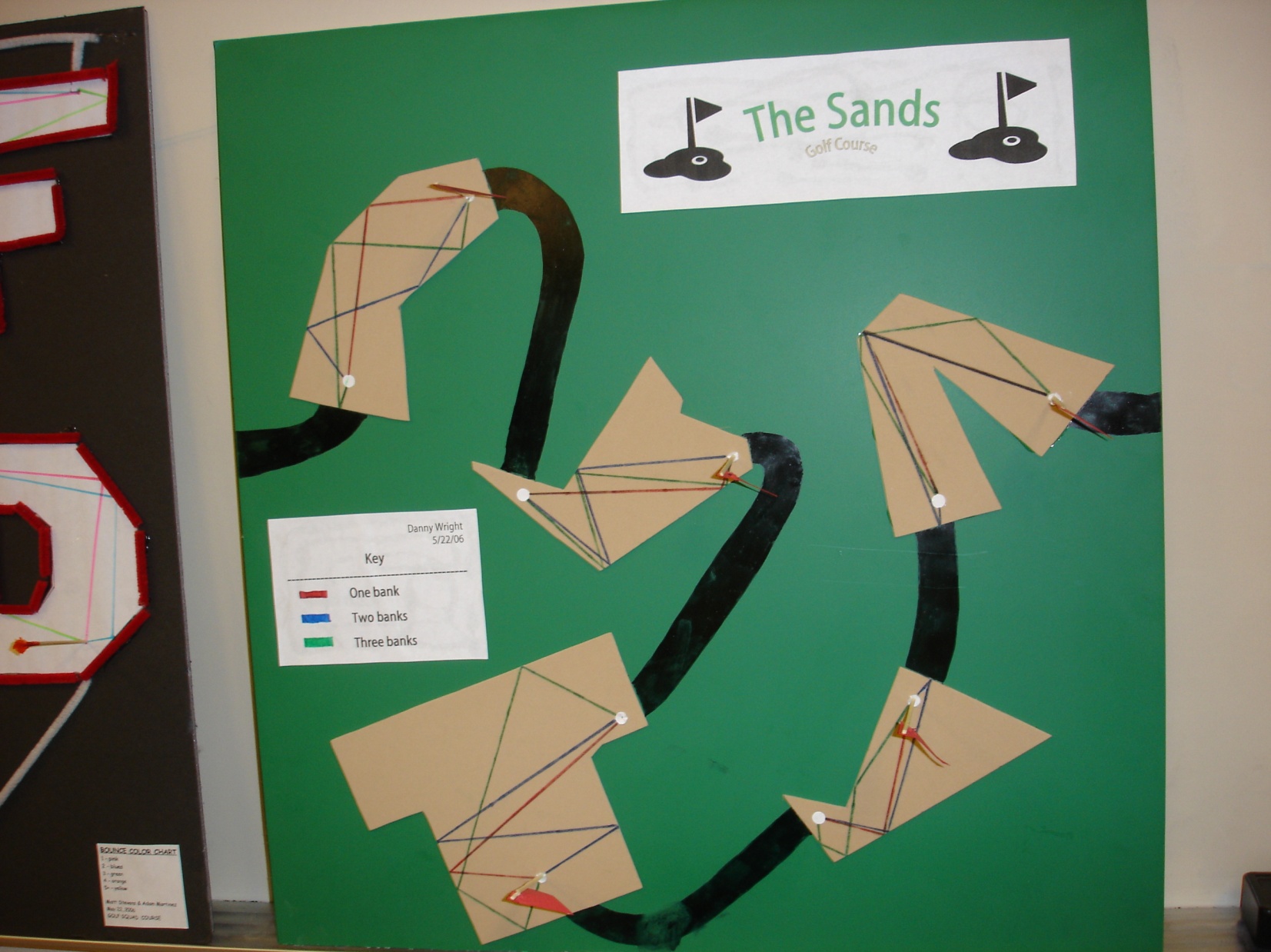
These paths for holes-in-one don’t work…





Resources and References:

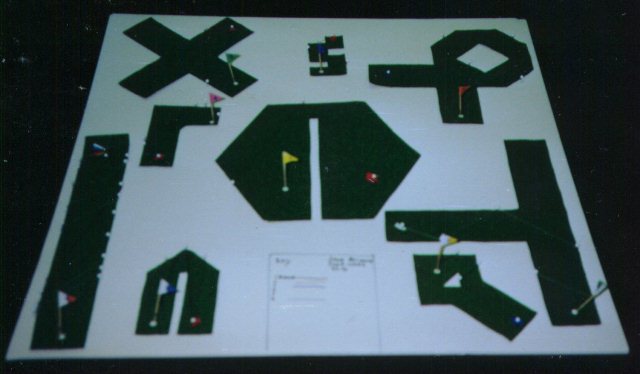
1. Powell, Nancy Norem, Mark Anderson & Stanley Winterroth, *Reflections on Miniature Golf*, The Mathematics Teacher, Oct 1994, 490-495
2. Directory of 37 Online Miniature Golf Games: <http://www.thepinballzone.net/games/online-mini-golf/>
3. Can you make a hole-in-one? <http://www.figurethis.org/challenges/c40/challenge.htm>
4. Mirror, Mirror: Reflections and Congruencies <http://www.andrews.edu/~calkins/math/webtexts/geom04.htm>
5. Mini golf activity <http://www.csiro.au/scope/activities/e29c02activity.htm>
6. Laws of Reflection <http://www.worsleyschool.net/science/files/amusement/reflection.html>
7. NCTM Illuminations:
   1. Paper Pool applet: <http://illuminations.nctm.org/ActivityDetail.aspx?id=28>
   2. Analyzing Numeric and Geometric Patterns of Paper Pool: <http://illuminations.nctm.org/LessonDetail.aspx?ID=L244>
8. A Mini-Golf Hole So Real It's Unreal: Intense reflection wins NYC art design contest for UB architect Joyce Hwang <http://www.buffalo.edu/news/9510>
9. Miniature golf books and articles <http://www.miniaturegolfer.com/minigolfbooks.html>
10. Hole-in-One! Activitiy <http://online.math.uh.edu/MiddleSchool/Modules/Module_4_Geometry_Spatial/Activities/Hole_One/UHGeometryHoleinOneTeacherNotes.pdf>
11. Math in your life – golf (video) <http://www.youtube.com/watch?v=26HqsQEmV-8>
12. Mathematics that Swings: the Math Behind Golf: lecture by Dr. Douglas N. Arnold (video – warning.. this is just a little over 1 hour long!) http://www.youtube.com/watch?v=e6v9ib-dOtg



## Nine-Hole Miniature Golf Scale Model Project



Use your imagination and design a three-dimensional 9-hole miniature golf course around a theme. The base of the complete model that you construct can be no larger than 22" x 28" (the size of a piece of poster board/foam core board). There is no limitation on materials for either the course or design features. Some suggestions for the surface include foam core board (lightweight and easy to work with), wood, corrugated cardboard, heavy poster board, etc.

Draw each hole (the same size as the final hole on the model) on a separate piece of paper that will allow you to

construct the paths for the holes-in-one. Use a red writing utensil to indicate a path when the ball bounces off of 1 wall, a blue writing utensil to indicate the path using 2 walls, a green line to indicate the path of the ball bouncing off of 3 walls, and other colors (make a key) for additional paths. This collection of drawings (all nine holes) should be checked when you’ve finished your holes, at least one week before your nine-hole golf course is due. After your holes have been checked, create your model and get your drawings ready to turn in with your final model.



The scale model must contain nine holes and a path between them. Each hole should include the ball placement, a flag at the hole with the hole number on it, and a minimum of 3 paths, color coded and include a color key on your course and you can be creative as to what you use to mark the paths on your model. Some suggestions for marking the paths include string, embroidery thread, yarn, markers, fabric paint (but practice with this first!) etc.

On the face of the model, include the following information:

\* your name and date

\* color key for your paths



\* a name for your golf course

j0238154Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Nine-Hole Miniature Golf Evaluation

|  |  |  |  |
| --- | --- | --- | --- |
| Category | Points Possible | Points Awarded | Comments |
| Scale Drawings (50 points) |  |  |  |
| Checked by the deadline | 5 |  |  |
| Accuracy of Constructions  (4 points per hole) | 36 |  |  |
| Neatly done and follows  required color codes | 9 |  |  |
| Scale Model (50 points) |  |  |  |
| Your name and date | 4 |  |  |
| Color key for your paths –  neat & complete | 5 |  |  |
| A name for your golf course -  creative and neatly displayed | 5 |  |  |
| Paths between holes | 9 |  |  |
| Accuracy of path placement  (3 points per hole) | 27 |  |  |
| 2 points per hole bonus for a path that banks off of 4 or more walls | 10  Extra Credit |  |  |
| Total -----> | 100 |  |  |